# Testing Arguments Using Indirect Truth Tables

Arguments with more than 3 simple propositions are very difficult to test using traditional truth tables. For example, an argument with four simple propositions requires 16 (2 x 2 x 2 x 2) lines, while an argument with five simple propositions requires 32 (2 x 2 x 2 x 2 x 2) lines. Things get (exponentially) worse the more simple propositions we have.

**Indirect truth tables** provide a solution to this problem; however, they require a bit more creativity/insight than do traditional truth tables (this makes them more challenging, but also a bit more fun!). Here’s how an indirect truth table works:

1. Translate the argument you are interested in into symbolic logic, using capital letters to represent each simple proposition, and the logical operators to represent the logical relationship between them. (This is the same as with traditional truth tables.)
2. Put a slash (“/”) between each premise, and a double slash (“//”) between the premises and the conclusion. (Again, this is the same as with traditional truth tables).
3. Every indirect truth tables begins with just ONE row. On this row, you should put an “F” under the main operator of the conclusion, and a “T” under the main operator of each premise.
4. You now work your way (from left-to-right) through the table and try to figure out what the truth values of *other* propositions are.
   1. You may have a few simple statements you don’t know the truth value of yet. Fill these in with whatever combination of “T” or “F” you want, though keep your end goal in mind (see the next step). Also, remember that the same letter needs to have the same truth value across the whole row.
5. Your goal is to show that it is *possible* to avoid a *contradiction.* A contradiction occurs when you are forced to assign a simple proposition *different truth values* on the same row of the truth table.
   1. If a row of a truth table is completed without a contradiction, then you have proved the argument **invalid.** You can stop now.
6. If a row does contain a contradiction, then you are done with this row. If (in step 4) you had to make any decisions about assigning F or T, you should now add a NEW row to the truth table that reflects a *different* way of assigning Ts and Fs.
7. If you have derived a contradiction in every row, and you cannot add any more rows, then the argument is **valid.**

## Example 1: A Valid Argument

Here is a sample valid argument: . We’ve already completed steps 1 and 2, so we’ll start at step 3, and make the premises true and the conclusion false:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | T |  |  |  |  | T |  |  |  |  | F |  |

Using our knowledge of the logical operators, we can know the following:

1. If a conditional is false, then the antecedent is true and the consequent false. By contrast, there are three ways of making it true.
2. If a disjunction is false, both sides are false. By contrast, there are three ways of making it true.
3. If a conjunction is true, both sides are true. By contrast, there are three ways of making it false.

Since is true, so are both ~B and C in premise 2. And since is false, both ~A and D are false in the conclusion.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | T |  |  | T |  | T | T |  | F |  | F | F |

And we can now figure out that A is true (since ~A is false). This allows us to fill out a bit more of the table:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| T | T |  |  | T |  | T | T |  | F | T | F | F |

But now we’ve run into a problem: according to the first premise, B *has* to be true (since making if false would make the conditional false). However, according to the second premise, B *has* to be false (since ~B is true):

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
| T | T | **T** |  | T | **F** | T | T |  | F | T | F | F |

This is a contradiction, and there is no way to add any more lines (after all, we *had* to assign each letter the truth value it currently has). So, the argument is valid—it is impossible to make the premises true and the conclusion false.

## Example 2: An Invalid Argument

Here is a subtly different argument, which is *invalid*: . We’ve already completed steps 1 and 2, so we’ll start at step 3, and make the premises true and the conclusion false:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  | T |  |  |  |  | T |  |  |  | F |  |

Since is true, we know both ~B and C are true, and that B is false:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  | T | F |  | T | F | T | T |  |  | F |  |

Since is true and B is false, we know that A is *also* false:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |
| F | T | F |  | T | F | T | T |  | F | F |  |

But we’ve now run into a problem: we can’t figure out the truth value of T. This means we could consider two possibilities: D is false or D is true. To do this, would need to add another line to our truth table (notice I’ve copied what we already know about the other letters):

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |
| F | T | F |  | T | F | T | T |  | F | F | T |
| F |  | F |  |  | F |  | T |  | F |  | F |

As it turns out, we don’t actually need two lines, since the first line is *consistent* (there’s no internal contradiction). Once we have shown a consistent line, we can confidently say “I’ve proved that this argument is INVALID—it is possible for all the premises to be true, and for the conclusion to still be false!” (There’s no need to continue doing the truth table after this point.) On the other hand, if the first row had been contradictory, we *would* have needed to use that second row. Remember these two rules:

1. An argument is invalid if AT LEAST ONE row on an indirect truth table is consistent.
2. An argument is valid if EVERY POSSIBLE row on an indirect truth table is contradictory.

## Using Indirect Truth Tables to Prove Propositions are Consistent

You can also use indirect truth tables to prove that a set of propositions is consistent (can all be true at the same time) or inconsistent (can’t all be true at the same time). The procedure is very similar to the one used to prove validity/invalidity, except with the following exceptions:

1. Instead of assuming that the premises are true and the conclusion is false, you assume that *all of the propositions are true.*
2. If there is at least one row that is consistent, then the propositions are consistent.
3. If EVERY POSSIBLE ROW is contradictory, the statements are inconsistent.

For example, suppose we want to know whether is consistent with . The indirect truth table would start like this:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | ~ |  | || |  |  |  |  |  |  |  |
|  | T |  |  | || |  | T |  |  |  |  |  |

In proposition 1, we can now figure out the truth values of A and B:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | ~ |  | || |  |  |  |  |  |  |  |
| T | T | T | F | || |  | T |  |  |  |  |  |

We now try making A true in the second proposition, and end up with this:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | ~ |  | || |  |  |  |  |  |  |  |
| T | T | T | **F** | || | T | T | **T** | T | T |  |  |

Uh-oh! There’s a contradiction! This means the two propositions are NOT consistent. The same thing would have happened if we had tried making B false in the second proposition (our truth table would have looked differently, but we would have had the same result):

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | ~ |  | || |  |  |  |  |  |  |  |
| **T** | T | T | F | || | **F** | T | F | F |  |  |  |

## Review Questions

1. Use indirect truth tables to determine whether the following arguments are valid or invalid:
2. Determine whether the following are true or false:
   1. When using indirect truth tables to prove validity/invalidity, you should begin by assuming that both the premises and the conclusion are TRUE.
   2. Indirect truth tables never have more than one line.
   3. If you discover a contradiction in a line of an indirect truth table, the argument is invalid.
   4. If both and your friend do an indirect truth table correctly, your answers will look exactly the same.
   5. If you discover that a line of an indirect truth table can be completed consistently, the argument is invalid.
   6. You should only add a new line to an indirect table if you can’t determine what the truth value of one or more simple propositions are/